DOCUMENT RESUME

ED 089 949 SE 016 913

AUTHOR	Bidwell, James K., Ed.
TITLE	An Introduction to the Minumum Performance Objectives for Mathematics Education in Michigan.
INSTITUTION	Michigan Council of Teachers of Mathematics.
PUB DATE	Feb 73
NOTE	25p.; Guidelines for Quality Mathematics Teaching; Monograph 2
AVAILABLE FROM	MCTM Publications Chairman, 2165 East Maple Road, Birmingham, Michigan 48008 ($1.00)
EDRS PRICE	MF-$0.75 HC-$1.85 PLUS POSTAGE
DESCRIPTORS	*Behavioral Objectives; Curriculum; Geometric Concepts; Instruction; *Mathematics Education; Number Concepts; Objectives; *State Standards

ABSTRACT

How and why the Minimum Performance Objectives were written, explanations of the objectives, how the objectives could be implemented, and a statement on the limitations of the objectives comprise the substance of this monograph. Illustrative objectives from "Minimal Performance Objectives of Mathematics Education in Michigan" are also included. For a complete description of the objectives, see SE 016 914. (JP)

THE MICHIGAN COUNCIL OF TEACHERS OF MATHEMATICS

M C T M

Guidelines for Quality Mathematics Teaching

A Monograph Series

AN INTRODUCTION TO THE MINIMUM PERFORMANCE OBJECTIVES FOR MATHEMATICS EDUCATION IN MICHIGAN

Monograph No. 2
February 1973

Additional copies of this monograph can be obtained at $1.00 each (prepaid) by writing to:

Horace L. Mourer
MCTM Publications Chairman
2165 E. Maple Road
Birmingham, MI 48008

Make checks payable to Michigan Council of Teachers of Mathematics (M.C.T.M.).

Other Monographs available from same address:

1. Classroom Proven Motivational Mathematics Games ($.60)

PREFACE

This monograph is the second in a series published by the Guidelines Committee of the Michigan Council of Teachers of Mathematics. This committee is jointly funded by the MCTM and the Michigan Education Association.

The Minimum Performance Objectives for Mathematics Education in Michigan has been approved by the State Board of Education. The document has been printed and distributed to all school districts in the state. This set of objectives will be the basis of future state assessment tests in mathematics. The impact of these minimal objectives on mathematics education in Michigan may be very great. Because of these facts the Guidelines Committee felt it necessary to provide an *Introduction* to the objectives to aid and inform teachers, supervisors, and administrators throughout the state.

This *Introduction* has *four* parts: how and why the objectives were written, explanations of the objectives, how the objectives could be implemented, and a statement on the limitations of the objectives. Throughout this monograph the *Minimum Performance Objectives for Mathematics Education in Michigan* will be referred to by the acronym MPOMEM. It is not necessary to have a copy of the MPOMEM in order to read this introductory monograph. Illustrative objectives from MPOMEM are included. Copies of the MPOMEM should be available in every school district for reference. Additional copies can be obtained by writing to:

Michigan Department of Education
General Services Area
P. O. Box 420
Lansing, MI 48902

This monograph was first outlined by the following committee: Richard Debelak (Iron Mountain), Jacqueline Dombroski (Petoskey), Geraldine Green (Royal Oak), Lou Henkel (Grand Rapids), Al Shulte (Pontiac), and James Bidwell (Mt. Pleasant). The first draft was written by Norma Berry (Grand Rapids), Lou Henkel, Al Shulte, and William Swart (Mt. Pleasant). The first draft was criticized by over seventy professionals at all levels throughout the state. The final draft incorporates many alterations suggested by the reviewers. The Guidelines Committee wishes to thank all of these persons for their time and guidance in producing this monograph. We hope that the MPOMEM will be better received and used throughout the state because of this introductory monograph.

James K. Bidwell
Editor

TABLE OF CONTENTS

INTRODUCTION

In October of 1971 a group of mathematics educators representing the Michigan Council of Teachers of Mathematics, the Detroit Area Council of Teachers of Mathematics, and the Greater Flint Council of Teachers of Mathematics was invited to meet with Dr. John W. Porter, Superintendent of the Michigan Department of Education, and members of his staff. The purpose of this meeting was to discuss the Department of Education's plans to develop minimal objectives on which state assessment tests could be based. This group presented to the Department of Education their position and some suggested specifications for the substance and format of a set of mathematical objectives. In particular, the group suggested:

1) That the format should be *sequential strands* of objectives, rather than objectives stratified according to grade level.
2) That the *developmental activities* so essential to the learner's understanding of mathematics should be included as *objectives*, rather than to include only the terminal objectives of computation.

As a result of this meeting, Dr. Porter asked that the mathematics educators of Michigan develop minimal objectives for school mathematics—objectives achieveable by all learners by the time they leave our schools. A team of thirty mathematics teachers from all levels (elementary, junior high, high school, college) was organized to write the objectives. It is significant that the twenty team members who did most of the writing, had an average of over eight years of K-8 classroom experience. In addition the group had much supervisory experience. Several two-day work sessions were held at which the objectives were written. Approximately 2000 man-hours went into the writing, total group discussion, and editing of the objectives. The team finished their work in December 1971.

During 1972 the set of minimal objectives was scrutinized at various conferences, critically read by mathematics educators, sent to selected school districts, and read by lay persons. The set of objectives have been approved by the State Board of Education as part of the accountability model for education in Michigan. They have been published by the Department of Education under the title *Minimum Performance Objectives for Mathematics Education in Michigan* (MPOMEM).

Why the Objectives Were Written

The Michigan Department of Education believed that written objectives were needed to provide a basis for the state education assessment program. The writing team, however, had considerable freedom to choose the substance and format of the objectives.

Behavioral objectives are not needed because our mathematical objectives have been vague and thus writing them behaviorally will make them clear, thereby improving instruction. That is nonsense. Perhaps objectives have been vague in some disciplines, but not in mathematics. We have known precisely what behavior we have wanted for stimuli, such as (1) "4 x 38", (2) "L x W x H," and (3) "1/3 x 3/5."

What has not been well understood is the role of developmental activities leading up to the desired terminal objectives. For example, work with diagrams and concrete objects has been frequently treated in a very few lessons under the assumption that such brief activity with the model makes a significant contribution to learning concepts and understanding operations. However, large numbers of learners require a great deal of work with the model for understanding to take place. The MPOMEM is based on a different assumption, namely, *that developmental activities are objectives in their own right.* Thus,

> the *OBJECTIVE* of using counters to multiply *precedes* the well-known objective of "4 x 38."
>
> the *OBJECTIVE* of placing cubic inch blocks in boxes to determine volume *precedes* the well-known objective "V = L x W x H."
>
> the *OBJECTIVE* of drawing a diagram depicting 1/3 x 3/5 precedes the well-known objective "1/3 x 3/5."

Mastery of the MPOMEM *developmental* objectives is viewed as important as and often a prerequisite for mastery of *terminal* objectives.

The writers hope that the style and substance of the objectives will have the following effects:

(1) That the *non-graded, sequential nature* of the operational objectives (addition, subtraction, multiplication, division) will promote *continuous progress instruction*. Thus, teachers of primary grades will give advanced work in fractions and decimals to those learners who are capable. Similarly, middle and upper grade teachers will help *some* learners to add, subtract, multiply, and divide whole numbers using objects, blocks, beads, and bundles. Instruction will become directed towards the individual and away from the large group.

(2) That *developmental instruction* with models, objects, and diagrams will require *the learner to perform* with those objects and diagrams, rather than to *observe* them as they are manipulated or drawn by the teacher. Thus, the learner will *make* bundles of ten as he learns

place value rather than to answer questions about pictures of bundles. He will *make* diagrams and cut-outs for fractions rather than to merely observe them in a book.

(3) That the *hands-on measurement* objectives will promote instruction that has the learner handling the measurement units and instruments in laboratory activities. Thus, a real (kitchen) scale will take the place of the pictures of a scale in a textbook. Real cubic inch blocks will replace textbook diagrams. A real thermometer will replace the diagrammatic one. Finally, each learner will *weigh, manipulate,* and *measure* with the units and instruments.

(4) That important objectives will be learned by *all* children in *two* and *three dimensional geometry.*

(5) That *pre-number concepts* will be an integral (planned) *part of the arithmetic program,* since the child does not necessarily enter the school ready to operate with numbers. Primary teachers will do considerably more diagnostic work to make sure, for instance, that the child can classify a set of objects according to color and to order a set of pictures according to height. More manipulative work (than is typical) will need to be done before the writing of numbers and formal arithmetic is begun.

The Nature of "Bench Marks" Within the Objectives

Each strand of objectives contains "bench marks" that separate the objectives into three categories: K-3, 4-6, and 7-9. These separations were provided by the writing team, *at the request of* the Department of Education, so the items for the fourth grade test could be chosen from the K-3 set and the seventh grade items from the 4-6 set. These separation marks are *solely* for the purpose of *state assessment,* and represent the best judgment of the writing team as to the level of mastery of most students in a typical instructional situation. They are not to be used as "time limits" for learners.

The writing team recognized a danger regarding these separations —the danger that teachers will consider those objectives appearing within a level (K-3, 4-6, 7-9) to be the responsibility of the teachers in that level *only. Just as there is no set of objectives that belong to any one grade level, there is no set of objectives that belong to any "cycle" of grade levels. The progress of any individual learner does not conform to any artificial grade structure.*

DESCRIPTIONS AND EXPLANATION OF OBJECTIVES

Organization of Objectives

The *Minimal Program Objectives for Mathematics Education in Michigan* (MPOMEM) is organized on a three-level basis. First, the objectives are grouped into five *Concept Areas:* Arithmetic, Measurement, Algebra, Geometry, Probability and Statistics. Second, within a concept area, the objectives are grouped into *strands.* For example, in the concept area of Arithmetic there are six strands: Number-Numeration; Whole Number; Common Fraction; Decimal Fraction; Integer; Ratio, Proportion, and Percent. Third, where appropriate, a strand is further subdivided into particular *topic sequences.* Thus, the whole number strand is further subdivided into four sequences: Addition, Subtraction, Multiplication, Division.

The chart on the following page lists for each concept area, strand, and sequence level the number of objectives within each bench-mark category. Note that there are not objectives at all levels in a given category. For example, there are no K-3 objectives in addition of fractions and there are no 7-9 objectives in addition of whole numbers.

The teacher will need to assume responsibility for objectives not assessed at his level of teaching. The sequences of objectives must be applied to the individual learner, not to a "grade level." The K-3 teacher will need to do introductory work with many 4-6 objectives; in fact, many learners will have already mastered many 4-6 objectives by the end of the third grade. Similarly, the junior high school teacher will probably find that not all of his class has command of the K-6 objectives. In fact, he will probably find it necessary to provide learning opportunities for many of the objectives.

NUMBER OF OBJECTIVES BY BENCH-MARK CATEGORY

CONCEPT AREA	STRAND	SEQUENCE	NUMBER OF OBJECTIVES K-3	4-6	7-9
Arithmetic	I. Numeration				
		A. Pre-Number	40	–	–
		B. Numeration	86	5	–
			126	5	0
	II. Whole Number				
		A. Addition	14	13	–
		B. Subtraction	16	5	–
		C. Multiplication	–	15	12
		D. Division	–	16	4
			30	49	16
	III. Common Fraction				
		A. Meaning	–	21	5
		B. Addition	–	7	5
		C. Subtraction	–	8	4
		D. Multiplication	–	4	6
			0	40	20
	IV. Decimal Fraction				
		A. Meaning	–	7	4
		B. Addition-Subtraction	–	10	–
		C. Multiplication	–	–	11
		D. Division	–	–	8
			0	17	23
	V. Integer		0	0	2
	VI. Ratio, Proportion, Percent		12	4	22
Total Arithmetic (366)			168	115	83
Measurement	I. Geometric				
		A. Linear	6	4	16
		B. Area	2	8	6
		C. Volume	5	3	3
		D. Angle	–	1	3
			13	16	28
	II. Non-Geometric				
		A. Time	7	4	2
		B. Money	3	6	3
		C. Temperature	2	2	1
		D. Weight	2	3	4
		E. Liquid	2	1	3
			16	16	13
Total Measurement (102)			29	32	41
Geometry	I. Identification				
		A. Shape	3	1	3
		B. Points and Lines	–	5	–
		C. Congruence	–	–	2
		D. Symmetry	–	1	1
			3	7	6
	II. Construction		–	5	3
Total Geometry (24)			3	12	9
Algebra (31)			1	9	21
Probability and Statistics (22)			5	5	12
Total Objectives (545)			228	163	154

A Typical Objective

Below is an objective taken from the MPOMEM. Notice that there are three parts: the *topic*, the *performance objective* itself, and *examples and comments.*

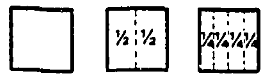
PERFORMANCE OBJECTIVES

PROPER FRACTIONS—LIKE DENOMINATORS, NO REGROUPING

1. Given a set of labeled fractional cut-out parts, the learner will demonstrate the result of subtracting two fractional numbers with like denominators of 2, 3, 4, 6, or 8 by arranging the appropriate parts, and then find and write the difference.

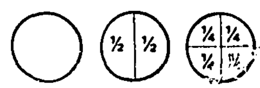
EXAMPLES AND COMMENTS

The fractional parts may be commercially prepared, teacher prepared, or student prepared. All the parts should be cut from *one common unit size*. The *unit whole* may be a rectangle or a circle, but once the unit is established, the *same* size and shape unit should be used for all the exercises.

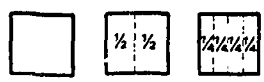


and so on

The learner should be able to use the cut outs interchangeably and this can only be done if all fractional parts originate from the *same unit whole.*

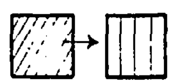


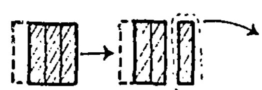
and so on



and so on

Problem: $\frac{3}{4} - \frac{1}{4} = ?$





The *topic* specifies the content area in which the objective lies. Frequently there are several objectives in a particular area, arranged in a learning sequence.

The *performance objective* indicates what the learner will be expected to do, and what materials (if any) he could use to achieve the objective. Since these objectives are considered minimal for all students, it is assumed that *all* learners will master the objective. Hence no criterion level of performance is given. However, the objective is stated in performance (or behavioral) terms.

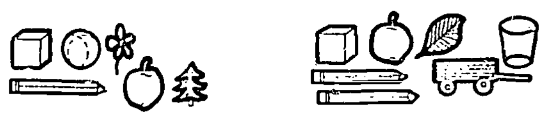
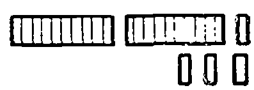
The *examples and comments* serve to illustrate the objective, to show possible procedures for developing the underlying ideas with learners, and to show some possible ways in which the learner may indicate that he has achieved mastery of the objective.

Kinds of Objectives

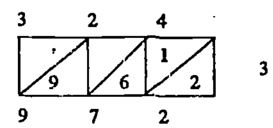
There are several kinds of objectives included in MPOMEM. Each type is briefly described and illustrated below.

Concept Objectives. Mathematics teaching should provide learners with an understanding of fundamental mathematical concepts. Mathematics as a discipline is concerned with the development of concepts which underlie the skill work. Since performance on a skill objective is much easier to assess than performance on a concept objective, many lists of mathematics objectives concentrate heavily on skills.

The writing group felt strongly that the understanding of key concepts is a part of the necessary mathematical background for all learners. In fact, concept objectives contribute greatly to computational ability. Thus, many concept objectives have been included, such as the following two examples. For contrast, the second of these objectives (9) is followed by its associated skill objective (10).

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
ADDITION—MEANING	
5. Given two sets of pictures, felt objects, and so on, totaling no more than 18 members, the learner will mentally form the union of the sets, and then orally and in writing tell the number sentence derived from that union.	"six plus seven equals thirteen" $6 + 7 = \boxed{13}$ 
ADDITION ALGORITHM—NO REGROUPING	
9. Given an addition exercise involving a *two digit* number plus a *one digit* number requiring no regrouping (carrying), the learner will demonstrate the process of addition using objects.	$\begin{array}{r} 21 \\ +\ 3 \\ \hline \end{array}$  Step I
10. Given addition exercises involving a *two digit* number plus a *one digit* number requiring no regrouping (carrying), the learner will find the sums with or without the use of aids.	 Step II Combine Objects and count them $\begin{array}{r} 21 \\ +\ 3 \\ \hline 24 \end{array}$ Step III

Skill Objectives. Mathematics teaching in the schools should result in proficiency in such areas as computation, equation solving, manipulating concrete materials, and analyzing problem situations. There are a large number of specific skills which learners need to master in order to use mathematics effectively in their lives. A typical skill objective is indicated below.

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
MULTIPLICATION—ALGORITHM	
19. Given a three-digit number, the learner will multiply it by a one-digit number.	Any of the methods below is acceptable: a) $324 \times 3 = \begin{array}{r}324\\ 324\\ +\ 324\\ \hline 972\end{array}$ b) Lattice  c) $\begin{array}{r}324\\ \times\ \ 3\end{array} = \begin{array}{r}300 + 20 + 4\\ \times\ 3 \qquad\qquad\\ \hline 900 + 60 + 12\end{array}$ $= 972$ d) $\begin{array}{r}324\\ \times\ \ 3\\ \hline 12\\ 60\\ 900\\ \hline 972\end{array}$ e) $\begin{array}{r}324\\ \times\ \ 3\\ \hline 972\end{array}$

"Hands-On" Objectives. A unique feature of the MPOMEM is the inclusion of many objectives which require the learner to exhibit his performance by "hands-on" manipulation of physical objects. This reflects the position of the writing group that many important mathematical objectives are not reducible to pencil and paper assessment or to measurement by selecting the correct answer on a multiple-choice item.

Manipulation is an important part of mathematics. For a learner to demonstrate proficiency with volume concepts, for example, it is necessary for him to have cubes and to use them to measure the volume of containers.

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
COMPARISON OF RECTANGULAR CONTAINERS WITH VOLUME UNITS	A box similar to a shoe box filled with cubic units.
4. Given a box filled with cubic units, the learner will determine by counting the number of cubic units used.	
5. Given a box and a supply of cubical blocks, the learner will determine the volume of the box in terms of the blocks.	

Developmental and Terminal Objectives. Many lists of objectives for mathematics include only *terminal* objectives—those objectives which indicate what a learner should be able to do at the end of a given period of instruction. The MPOMEM includes many terminal objectives. The writers went far beyond this, however, by indicating *developmental* objectives as well. The developmental objectives are necessary prerequisites for the desired terminal behaviors. They represent various stages along the way to mastery of the terminal objectives. These developmental objectives are arranged in a sound learning sequence, so that each developmental objective contributes to the next, culminating in the desired terminal objective.

It should be understood that developmental objectives related to a particular terminal objective may be taught or studied years apart. For instance in the given example, solving a simple linear equation, the developmental objectives begin in the primary grades, with the terminal objective to be achieved by typical students in grades 7-9. In the example provided, objectives 1, 2, 3, 4, and 11 are developmental objectives, and 13 is the related terminal objective.

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
EQUATIONS	
1. Given a statement of equality involving addition or subtraction facts and a place-holder for the sum or difference, the learner will supply the sum or difference.	1. $5 + 3 = \square$ 2. $5 - 3 = \square$

BENCH MARK $\frac{\text{K-3}}{\text{4-6}}$

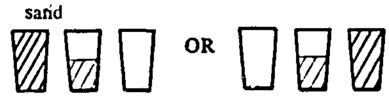
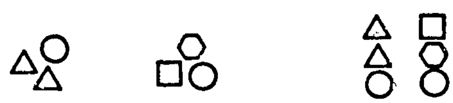
2. Given a statement of equality involving addition, subtraction, or multiplication facts and containing a place-holder or letter, the learner will find the missing number.	1. $5 + \square = 7$ 2. $\square + 6 = 8$ 3. $7 - \square = 3$ 4. $\square - 6 = 2$ 5. $5 \times 4 = \square$ 6. $5 \times \square = 25$ 7. $\square \times 3 = 12$	8. $8 + 7 = n$ 9. $6 + n = 11$ 10. $n - 3 = 10$ 11. $n = 9 - 7$ 12. $5 - n = 3$ 13. $n - 8 = 4$ 14. $5n = 20$
3. Given two numerical phrases, the learner will compare correctly the expressions by using $=$ or $\neq$.	$5 + 9 \bigcirc 14$ $5 \times 3 \bigcirc 16$ $8 + 7 \bigcirc 2 \times 3$	

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
SYMBOLS	
4. Given a pair of whole numbers or number phrases less than 1000, the learner will supply the appropriate symbol of equality or inequality, $<$ or $=$ or $>$.	1. 3 ◯ 4 2. $6 + 2$ ◯ 3 3. $5 + 7$ ◯ 12 4. $8 + 3$ ◯ $7 + 5$ 5. 517 ◯ 240
EQUATIONS	
11. Given an equation involving addition, subtraction, multiplication, or division of whole numbers and involving a variable, the learner will find the value of the variable.	Methods: 1. Trial and error 2. Intuition 3. Number facts 4. Inverse operations Examples 1. $8 \div 4 = n$ 2. $24 \div n = 6$ 3. $n \div 8 = 9$ 4. $\frac{x}{3} = 6$ 5. $18 \div x = 3$ 6. $8 + 7 = n$ 7. $6 + n = 11$ 8. $n - 3 = 10$ 9. $n - 9 = 7$ 10. $5 - n = 3$ 11. $n - 8 = 4$ 12. $5n = 20$
13. Given a linear equation of the form $ax \pm b = c$, where a, b, c, and x are whole numbers, and the solution is a whole number, the learner will be able to find the solution.	Methods: 1. Trial and error 2. Balance beam 3. Axioms of equality may be used. 4. The learner may use the "cover-up" technique. Examples: 1. $4x + 8 = 12$ 2. $3y - 6 = 9$ 3. $11n + 10 = 21$ 4. $16 - 5a = 11$

Pre-Number Objectives. An important and unique feature of the MPOMEM is the pre-number strand. The pre-number objectives underlie and precede the numerical work. If a child has not mastered these objectives, he will be much less effective in the related numerical computations and his understanding of the related number concepts will be weakened. Unfortunately, these prerequisite skills and concepts have not been set forth previously in a comprehensive manner as a guide to beginning mathematics instruction. The compilation of these pre-number objectives in the MPOMEM is a landmark in the writing of mathematics objectives.

Teachers of kindergarten and primary grades should provide experiences that will enable most of their learners to master the pre-number objectives before proceeding with the development of numerical and computational work. Older children who did not achieve mastery will need further work with pre-number activities.

Five objectives from the pre-number strand are included below.

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS
CLASSIFICATION—ATTRIBUTES	
8. Given a set of two or more (a) circle (b) triangle (c) square or rectangle shaped objects and shown an object of one of those specific shapes, the learner will pick up an object which is the same shape.	(a) ball (b) triangularly folded paper hat (c) box or book
ORDERING	
16. Given a set of three clear plastic drinking glasses, one filled with sand; one empty; and one half filled with sand, the learner will arrange the glasses from "full to empty."	The "full to empty" can apply either to the sand or the air. *Clear* glasses are specified because the children must be able to see through the containers. The *plastic* glass material is suggested as a safety factor to prevent the injury from broken tumblers. sand 
CLASSIFICATION—ATTRIBUTES	
20. Given a set of ten objects of varying shapes and textures, the learner will pick out objects having specific combinations of two attributes.	Given: Shapes: squares, circles, triangles Textures: rough (sandpaper) smooth (velvet) Possible Combinations: rough triangle shape, smooth circle shape, etc.
NUMBER MEANING	
33. Given a collection of from one to nine small objects and a length of yarn or string, the learner will place *some* of the objects inside the closed curve formed by the string.	
38. Given two equivalent sets of small objects (2 to 5 members), the learner will demonstrate a one-to-one matching by physically associating the objects of one set with the objects of the second set.	Child moves objects  Set S Set T

Relationship of Objectives to the Instructional Program

The objectives contained in the MPOMEM provide a *lower bound* for the instructional program. These objectives were judged to be so important that *all* students in the state should master them. However, they cannot be considered as the total instructional program. Many learners can and should perform well beyond the minimal objectives listed. The mathematics curriculum should not be limited to just the teaching of these minimal objectives.

The objectives (and the learner's progress through them) will help define the high priority topics in the curriculum. However, considerable time in any year should be given to mathematics which will be valuable for the learners, but which will not be assessed as part of the objectives (at least at that level.)

Additionally a teacher should not think "I'll cover objective 1 today, objective 2 Tuesday, objective 3 Wednesday and Thursday." Usually learners will need the entire school year (or years) to master a list of sequential objectives.

The Difference Between Objectives and Lessons

Teachers should not think of the objectives as abbreviated lesson plans. To illustrate the difference between objectives and lessons, and also to emphasize their interrelationship, a set of consecutive objectives and a lesson plan outline based on those objectives are included in the following pages. The outline assumes traditional large group instruction and testing on sequential days. Neither of these conditions are forced by the objectives; alternate teaching formats exist and are recommended for most of the objectives. In some cases consecutive objectives would be mastered months, or even years apart.

A SET OF CONSECUTIVE OBJECTIVES

PERFORMANCE OBJECTIVES	EXAMPLES AND COMMENTS	
2. Given a statement of equality involving addition, subtraction, or multiplication facts and containing a placeholder or letter, the learner will find the missing number.	1. $5 + \square = 7$ 2. $\square + 6 = 8$ 3. $7 - \square = 3$ 4. $\square - 6 = 2$ 5. $5 \times 4 = \square$ 6. $5 \times \square = 25$ 7. $\square \times 3 = 12$	8. $8 + 7 = n$ 9. $6 + n = 11$ 10. $n - 3 = 10$ 11. $n = 9 - 7$ 12. $5 - n = 3$ 13. $n - 8 = 4$ 14. $5n = 20$
3. Given two numerical phrases, the learner will compare correctly the expressions by using = or ≠.	$5 + 9 \bigcirc 14$ $5 \times 3 \bigcirc 16$ $8 + 7 \bigcirc 2 \times 3$	
SYMBOLS		
4. Given a pair of whole numbers or number phrases less than 1000, the learner will supply the appropriate symbol of equality or inequality, $<$ or $=$ or $>$.	1. $3 \bigcirc 4$ 2. $6 + 2 \bigcirc 3$ 3. $5 + 7 \bigcirc 12$ 4. $8 + 3 \bigcirc 7 + 5$ 5. $517 \bigcirc 240$	
EQUATIONS		
5. Given an equation involving one or zero, the learner will complete the sentence.	1. $5 \times 0 = \square$ 2. $7 \times \square = 0$ 3. $7 \times 1 = \square$	4. $8 \times \square = 8$ 5. $6 + 0 \quad \square$ 6. $7 + \square = 7$
6. Given a number sentence, the learner will indicate whether the sentence is true or false.	1. $4 \times 7 \overset{?}{=} 29$	2. $3 + 12 \overset{?}{=} 15$

A POSSIBLE LESSON PLAN OUTLINE

Day 1: 1) Review of placeholders used to express a sum or difference.
Examples: (a) $5 + 4 = \square$
(b) $7 - 3 = \square$

2) Oral work on supplying missing numbers in a sentence.
Examples: (a) Five plus some number makes eight. What is the number?
(b) The product of two numbers is 15. One of the numbers is 3. What is the other number?

3) Sentences with placeholders—class solves examples at board.
Examples: (a) $5 + \square = 8$
(b) $3 \times \square = 15$

4) Each learner works 15 exercises of the type indicated in Examples 1-7, Objective 2.

Day 2: 1) Exercises from previous day are reviewed.

2) Three exercises are selected; learners are asked to make up stories which fit these exercises.

3) Learners share their stories.

4) Mental Arithmetic: Practice on addition and multiplication facts. Conducted as a game—boys against girls.

Day 3: 1) Teacher leads discussion: "What do we mean by equal?" "When are two expressions about numbers equal?" Conclude that two expressions about numbers are equal when they name the same number.

2) The equal sign ($=$) is reviewed.

3) Learners are asked if they know how to indicate "not equal" ($\neq$).

4) Use of the slash (/) to indicate "not" something in mathematics compares with slash on international highway markings. For example, $\boxed{P}$ means "parking permitted"; $\boxed{\cancel{P}}$ means "no parking".

5) Learners work 20 exercises of the type shown in Objective 3.

Day 4: 1) Review of the word "inequality" and the symbols for "greater than" and "less than".

2) Exercises where learners indicate whether one expression is $<$, $=$, or $>$ a second expression. The teacher states the two expressions to be compared, or writes them on the chalkboard. The learners hold up index and middle fingers of left hand to simulate "less than", index fingers of each hand parallel to simulate "equal", or index and middle fingers of right hand to simulate "greater than." This feedback allows the teacher to correct student misconceptions immediately.

3) The previous day's homework is reviewed.
4) Students work 15 exercises of the form shown in Objective 4.

Day 5: 1) Teacher says, "I'm thinking of a number. If I multiply 5 by the number, I get 5. If I multiply 3 by the number, I get three. If I multiply 427 by the number, I get 427. What number am I thinking of? Discuss the multiplication property of one, after the number is correctly identified.
2) Teacher asks sequentially: "What number can I multiply 8 by to get 0?" "What number can I multiply 12 by to get 0?" "What number can I multiply 20 by to get 0?" "What happens when I multiply a number by 0?"
3) Given exercises as in Objective 5. Correct papers in class and discuss errors.
4) Stress the difference between multiplying by 0 and multiplying by 1.
5) Use the properties of 0 and 1 to fill in the first two rows and columns of the multiplication table. Explore other patterns in the multiplication table.

Day 6: 1) Hand out a ditto sheet with 30 English sentences. Students mark each sentence as "True", "False", or "Can't tell".
2) After completing the activity in (1), discuss what would be needed to change a "can't tell" example into either a "true" or "false" one.

Day 7: 1) Hand out a sheet containing 20 exercises as in Objective 6. Students should
(a) Tell whether a given expression is true or false;
(b) Change the false expressions so that they are true.
2) Discuss the changes which students made and whether these, in fact, produce true sentences.

Day 8: As a diversion, the class works with the Sieve of Etatosthenes for numbers from 1-100.

Day 9: 1) Give 2 or 3 examples of sentences with placeholders (as on day 1).
2) Teacher says: "Suppose I use the letter *n* instead of a box to represent the number we don't know. Then $5 + n = 12$ means, "What number do I add to 5 to get 12? What number could we use in place of *n* to make the sentence true?"
3) Learners work on 20 exercises like Examples 8-14 in Objective 2. Given the sentence and a number to substitute for *n*, they are to tell if the resulting sentence is true or false.

IMPLEMENTING MPOMEM LOCALLY

The process of implementing the MPOMEM is of vital importance and a distinct challenge to the educator who wishes to derive its full benefits within the school district. Its very strengths will at times present momentary obstacles to quick and easy implementation. Non-graded strands of sequential objectives provide a basis for precise diagnosis and prescription of minimal student needs on an individual basis. Simultaneously, the concerned educator must devise instruments for such diagnosis and prescription. Instructional methods and materials must be developed which will meet individual needs. Much creativity is needed if classrooms and equipment are not oriented to activity learning.

MPOMEM as a Focal Point

When implementing the MPOMEM, the educator must decide which objectives are pertinent and how learning can be assured, measured, and duly recorded for each learner. The MPOMEM provides school districts with a sound realistic minimal program of concepts and skills that every child can master during his school experience. Part of the educator's task is to build, from this nucleus, a quality program which provides the full range of mathematical opportunity to which the majority of Michigan school children should have access. The MPOMEM provides a focal point for minimal learning as well as for an acceleration program if one is desired.

An inherent danger in focusing on minimums is that the casual observer may regard the minimums as *ready-made, complete programs*. Such an attitude could destroy attempts to provide a quality mathematics program within the school system. It is important to realize that *expansions* of topics are expected for *most* children. A wealth of enrichment opportunities is desirable for those children who master the "basics" with ease.

The organization of the MPOMEM is flexible, yet sufficiently specific to be used in many ways. The following are some recommended uses for the MPOMEM:

1) As a basis for *evaluating present programs* at the local level.

The evaluation could be done by:

a) supervisory personnel at the district level,

b) administrative personnel in each building,

c) teachers in each classroom.

The evaluation should include:

a) content emphasis: "Are children who have difficulty with essentials wasting time with non-essentials?"

b) Topic presentation: "Are all essential topics being taught, and are they taught to appropriate levels?"

c) Prerequisite skills: "Does the learner know everything necessary to learn this new topic?"

d) Individual prescription: "Is this student learning at his level or is he being bored (frustrated)?"

2) As a basis for establishing some continuity and consistency in a *minimum program* for *all* children that

   a) allows for wide varieties of approaches for mastery.

   b) permits extension beyond the minimum level of content.

3) As a basis for *teacher training* needs

   a) at the preservice level in colleges.

   b) at the inservice level within a school district.

The Management System

Once the objectives for local education have been established, it becomes necessary to devise effective methods for achieving them. A collection of such methods is commonly called a *management system*. The components of a management system usually include the following: 1) Formulation of Objectives; 2) Diagnostic Procedures; 3) Placement and Prescription; 4) Strategies for Instruction (the teaching/learning situation), which includes such things as grouping techniques, methods of presentation, motivational techniques, selection of materials, and utilization of personnel; 5) Measurement (Testing), followed by Re-Teaching, Review, and Maintenance; 6) Record-Keeping; and 7) Reporting System.

Responsibility for the development of such a management system may be in the hands of the individual classroom teacher, the building administrator, or a subject area supervisor. Regardless of who develops it, the MPOMEM is an obvious framework for building management systems.

The fourth part of the management system, strategies for instruction, is important enough to warrant elaboration, particularly the spiral method of presentation.

Spiral Development in Text Series

It should be recognized that no particular mathematical topic is treated successfully by spending ten days on the topic and then leaving it for all time. Most *elementary text series* incorporate a spiral development, whereby a particular topic is studied at a variety of levels, with the depth and sophistication of the approach increasing with each new look at the topic. The following four level plan (over a four year or longer period) is typical.

1) *Introduction or Exposure.* In the first exposure period, the learner is given some exploratory and familiarizing experiences. *No mastery* of any kind is assumed.

2) *Development Work.* The second time a topic is presented the introductory notions are reviewed, and the developmental work is carried out. There should be *no* pressure for rapid calculation or skill. Emphasis should be more *on the process* than on the answer.

3) *Mastery.* During the third exposure the student should work for mastery of the topic. The process will be briefly reviewed, but the major emphasis is on getting an answer as correctly and efficiently, as the learner's ability allows.

4) *Maintenance.* Following the work on mastery, a topic will be touched upon in succeeding grades as maintenance — keeping the skill functioning at a high level for each learner.

It is important for the teacher to realize that text series are written in this fashion. A teacher can make more efficient use of a text if he recognizes the level at which the text approaches the topic. For most learners development will precede mastery by months or years. Such learners should not be pushed for mastery on each topic. The MPOMEM does not assume any rigid time line for a class or even individual learners. The four levels of development of a topic will overlap for many learners. The rate of progress through the levels will vary with the individual learner and cannot be locked into a particular grade or textbook.

The Classroom Teacher and the MPOMEM

The *classroom teacher* will finally determine the effectiveness of the objectives of the school system. While administrative support is certainly desirable, the classroom teacher should be able to implement the MPOMEM by adhering to the following steps. They are suggested regardless of whether implementation is initiated by mathematics supervisory personnel, building supervisors, or the classroom teacher.

1) Assessment of current student performance in terms of the objectives.

2) Determination of instructional levels and topics on the basis of assessment.

3) Selection and correlation of instructional materials to the concepts and skills to be taught.

4) Provision for a meaningful and efficient record-keeping system. Selection can be made from several types:

a) Record of Individual Progress (Cumulative)

This shows the learner's exposure to and the mastery of given objectives. This record allows a teacher to see deficiencies and strengths of individual learners at a glance and serves as a basis for planning instruction, maintenance, and review for each learner.

It accompanies the learner as he moves from teacher to teacher and from school to school. Each teacher records the learner's progress.

A sample scheme for individual records is provided by the sequences in the MPOMEM. Suppose a sequence strand is numbered and filled in for a learner as follows:

Addition of	1	2	3	4	5	6	7	8	9
Whole Numbers	⊗	⊗	⊗	○	⊗	○	○		

The sample code can be interpreted: ○ indicates that extensive exposure has been given to the learner; ⊗ indicates that a learner has mastered the objective. In a graded school, a numeral in place of the X could be used to indicate the grade when the objective is achieved.

b) Record of Individual Progress (Short Term)

This could be a check list form, for use during a given unit or marking period. This sort of record provides an opportunity to involve learners directly in goal-seeking, as well as providing parents with frequent and pertinent information on progress. Each child could keep his own record.

c) Class Profile

This enables a teacher to assess the needs of the group as a whole and to plan efficient small group instruction or individual activities.

5) Evaluation of learner progress (growth) in terms of the objectives. A complete testing program would include pretests and posttests with items keyed to specific objectives.

6) *Evaluation of instructional program and teaching methods in terms* of learner growth.

LIMITATIONS OF THE OBJECTIVES

As has been clearly stated earlier, the MPOMEM is *not* a mathematics *curriculum*. It is not even an adequate list of objectives for a comprehensive program. In fact, most students will learn more than the basic set of objectives by just using the minimal program prescribed in most adopted commercial programs in the state.

The writers of the objectives purposely omitted many very desirable objectives because their charge was to produce a *minimal list* that could be achieved by all the students in the state. Among the omissions were mathematics vocabulary, some field properties (commutative and associative laws), division of fractional numbers, problem solving techniques, applications of skills in real life, and objectives dealing with critical thinking and creative thinking. Many topics that are essential background for high school mathematics classes are either omitted or only introduced.

Hopefully it should be clear that the MPOMEM is *not* a mathematics curriculum and does not provide adequate objectives for a quality mathematics program for grades K-9. It is the *responsibility of the local school district to determine suitable performance objectives beyond the minimal sequences* in order to provide a comprehensive program that will satisfy the needs of their learners. The minimal sequences should be lengthened and broadened and new sequences need to be added.

Even within the minimal sequences presented in the MPOMEM, the treatment is not complete. Alternate objectives may be preferable and some sequences themselves may need modification, probably through the insertion of intermediate objectives. Some of the terminology may be different from that used in some textbook series and hence unfamiliar to teachers. Within the section marked "comments and examples" a limited number of modes of presentation have been suggested. Many alternatives exist that will also provide for effective learning. In particular, there are many concrete aids, games, and laboratory activities that would be appropriate. The *printed* nature of the comments should not limit the imaginative teacher.

These limitations do not detract from the original premises under which the MPOMEM was written. It does provide a basis for an adequate minimal program of mathematical learning by all students and a foundation on which to build a complete program of mathematics education.

The following were members of the Guidelines Committee for Quality Mathematics Teaching during the development of this monograph:

Maja S. Barr	Saginaw
Richard Debelak	Iron Mountain
Thersea Denman	Detroit
Ronald Dirkse	Stevensville
Jacqueline Dombroski	Petoskey
Lou Henkel	Grand Rapids
Jean Houghton	Weidman
Phillip S. Jones	Ann Arbor
Patricia Kesterke	Alpena
Daniel Korman	West Branch
Evelyn Kozar	Detroit
Donald McPhee	Saginaw
Bea Munro	Ann Arbor
Richard Mignery	Lansing
Tamara Sihon	Port Huron
Geri Westover	Bay City
James K. Bidwell (Chairman)	Mt. Pleasant

Your comments and criticisms of this monograph as well as suggestions for other monographs (or manuscripts for them) can be sent to

James K. Bidwell
Department of Mathematics
Central Michigan University
Mt. Pleasant, MI 48858

MICHIGAN COUNCIL OF TEACHERS OF MATHEMATICS
2100 PONTIAC LAKE ROAD
PONTIAC, MICHIGAN 48054

Non-Profit Org.
BULK RATE
PAID
PERMIT No. 166
Pontiac, Mich.